

# Multiparameter Mapping of Polymer Properties for Fast Thermolysis of Powder Moldings

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**ABSTRACT:** Ceramic or metal injection molding involves the shaping of a crowded suspension of particles in a polymer that is later removed by thermolysis. During heating, degradation products dissolve in the parent polymer and diffuse to the free surfaces where evaporation occurs. If heated too rapidly in the initial stage, the solution at the center of the molding boils and a bubble forms. This article explores the implications of a multiparameter model that predicts the highest permissible heating rate in terms of polymer properties and maps the outcome. The aim is to develop criteria for the deliberate synthesis of thermally labile copolymers from a knowledge of the desirable property combinations. © 1998 John Wiley & Sons, Inc. *J Appl Polym Sci* 70: 545–557, 1998

**Key words:** diffusion; thermal degradation; mathematical modeling; ceramic processing

## INTRODUCTION

In metal and ceramic injection molding, crowded suspensions containing 50–70 vol % of powder in a polymer vehicle are prepared by high shear mixing prior to forming. The vehicle, which has a transient role, is removed, usually by slow heating, before sintering the powder assembly that should retain its shape and be free from defects.<sup>1–3</sup> Every stage in this sequence depends on different properties of the polymer. The most difficult stage, particularly for large sections (>15 mm), is the removal of the vehicle by controlled thermolysis. Procedures have been established to use gravimetric control to determine the rate of heating<sup>4</sup> and to use composite thermogravimetry to select combinations of organic media to give a steady weight loss on heating,<sup>5,6</sup> but

these approaches address only the rate at which volatile degradation products are evolved. They do not consider the mass transport kinetics in a large section molding.

Low molecular weight organic vehicles are lost by evaporation from the surface of moldings,<sup>7</sup> and the use of oxidative atmospheres introduces a surface reaction with shrinking core kinetics.<sup>8</sup> Often polymer degradation takes place in a nonoxidizing atmosphere.<sup>1</sup> Thermal degradation accompanies these processes and, when the temperature is above the ceiling temperature, takes place uniformly throughout the section liberating low molecular weight products of decomposition. These dissolve in the parent polymer, and evaporation from the surface produces a concentration profile that is the driving force for outward diffusion of products in solution in the continuous phase. If the temperature rises above the boiling point of the solution at the center of the section, where the low molecular weight component is most concentrated, boiling occurs and a defect is produced.

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This occurs in the critical stage when all pores are filled by organic vehicle. Later, as the polymer fraction reduces, continuous porosity forms and gas transport paths become available. The critical stage has been recognized and modeled for steady state<sup>9</sup> and unsteady state diffusion.<sup>10</sup> The model has been extended to consider the development of various porosity configurations,<sup>11</sup> for the principal geometries,<sup>12</sup> for gas transport in a porous outer layer,<sup>13</sup> and for a surrounding powder bed.<sup>14</sup> The model has been tested experimentally using coarse alumina powder and poly( $\alpha$ -methylstyrene) at atmospheric pressure<sup>10,12</sup> and at elevated pressures.<sup>15</sup> In these models, polymers that undergo thermal decomposition to monomer are considered because the existence of more than one degradation product complicates the analysis immensely. The diffusion coefficient for each product is affected by the concentration of every other product, and multicomponent interaction parameters would be needed in deducing the boiling points. Several polymers that decompose to monomer, such as the short ester group methacrylates, poly( $\alpha$ -methylstyrene), or polyoxymethylene can be used in ceramic processing.

An initial exploration of the implications of the model has been conducted to rank the influence of monomer and polymer properties.<sup>16</sup> This was achieved by holding a set of default parameters corresponding to poly( $\alpha$ -methylstyrene) and its monomer and varying each one individually over the range of values that is consistent with known polymers. This type of sensitivity analysis is restrictive in that it does not disclose heating rate maxima resulting from simultaneous variation in all parameters.

From this sensitivity<sup>16</sup> analysis, the parameters that have been found to influence the critical heating rate strongly fall into three groups: (1) those controlling diffusion coefficient as calculated from free volume theory,<sup>17</sup> namely pre-exponential coefficient  $D_0$ , activation energy for diffusion  $E_d$ ; (2) those that control the shape of the thermogravimetric curve, namely specific rate constant  $K_0$  and activation energy  $E$ ; and (3) the parameters that influence the boiling point of the monomer, namely  $\Delta H_{\text{vap}}$  and  $i$  the logarithm of the pre-exponential coefficient in the Clausius–Clapeyron relationship.

These parameters give a preliminary indication of how the synthesis of thermally labile polymers for ceramic processing might be approached. Thus, the size of the degradation product molecule and its polarity influence  $\Delta H_{\text{vap}}$  and could be influenced by introducing weak links to control

chain scission sites. The problem is that, while increasing the diffusant molecular size increases the boiling point it is likely to reduce  $D_0$ , another sensitive parameter; and so, a delicate balance of “design features” for the polymer emerge.

The aim of this article is to explore the effects of varying, in combination, the parameters that have been found to be influential. Data so generated, which are potentially vast, should be capable of being comprehended and interpreted on a physical basis. In the past, polymers selected for ceramic processing have tended to be available production grades. They have not been synthesized for their thermal lability; indeed, quite the reverse. There is scope for advances in polymer synthesis to be applied to ceramic processing, and this work is intended to pave the way for this.

## THEORY

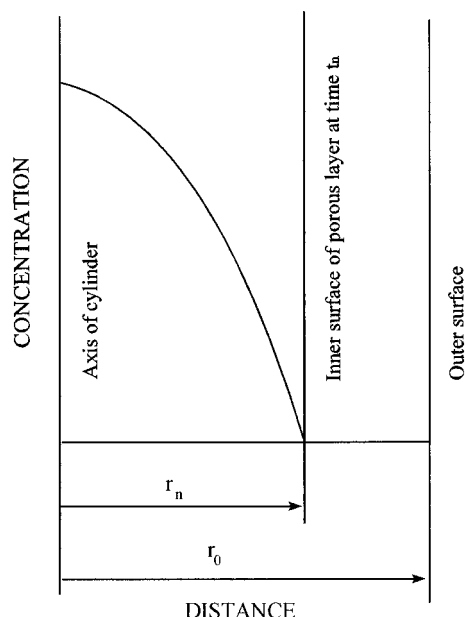
An infinite cylinder of radius  $r_0$  containing an inert filler, such as a ceramic powder, at volume fraction  $V_c$ , is considered. A single polymer that saturates the pore space, which consequently has a volume fraction  $1 - V_c$ , is assumed to degrade exclusively to the monomer. The molding is heated at a constant heating rate,  $Z$ , that is sufficiently low for the temperature gradient caused by transient heat transfer to be neglected. This condition holds for the experimental validation of the model where the heating rates are low or where high uniform rates of heating can be achieved (e.g., microwave-assisted). The mathematical model determines the critical linear heating rate,  $Z_c$ , at which the peak vapor pressure along the axis of the cylinder is below the ambient pressure throughout the heating process. Thus, defects due to boiling of the monomer in solution in the parent polymer do not occur.

As the polymer degrades, it is considered to recede in the interparticle pore space<sup>11</sup> (Fig. 1). Assuming that the initial radius of the cylinder is  $r_0$ , the radius  $r_n$ , of the shrinking undegraded core at some subsequent time  $t_n$ , is given by

$$r_n = r_0 h^{1/2} \quad (1)$$

where  $h$ , the remainder weight fraction of polymer, is defined by

$$h = \exp \left\{ -\frac{K_0 R T^2 \exp(-E/RT)}{ZE} \times \left[ 1 - \frac{2RT}{E} + \frac{6R^2 T^2}{E^2} \right] \right\} \quad (2)$$



**Figure 1** Schematic diagram of the shrinking undegraded core model for a cylinder with initial radius  $r_0$ .

(see, e.g., Evans and colleagues<sup>10</sup>). Descriptions of the symbols used in eq. (2) and throughout the article are given in the Nomenclature section.

The monomer concentration within the undegraded cylindrical core satisfies the partial differential equation

$$\frac{\partial C}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( rD \frac{\partial C}{\partial r} \right) + \dot{Q}; \quad 0 < r < r_n; \quad t > t_n \quad (3)$$

in which  $\dot{Q}$ , the rate of generation of monomer based on unit volume of suspension, is given by

$$\dot{Q} = \dot{q} \rho_p (1 - V_c) \quad (4)$$

where

$$\dot{q} = K_0 h \exp(-E/RT) \quad (5)$$

is the rate of production of monomer based on the mass of polymer. The diffusion coefficient,  $D$ , for monomer in the unfilled polymer, varies with temperature,  $T$ , and with concentration,  $C$ . The free volume theory of diffusion<sup>17</sup> can be used to define  $D$  over a wide range of values of  $T$  and  $C$  from the expression

$$D = D_{01}(1 - \phi)^2(1 - 2\chi\phi)\exp\{-[W_1V_1(0) + W_2\xi V_2(0)]/(V_f/\omega)\} \quad (6)$$

for diffusion in the continuous phase, where  $\phi$ , the volume fraction of monomer in polymer, is given by

$$\phi = W_1V_1/(W_1V_1 + W_2V_2) \quad (7)$$

and the diffusion constant  $D_{01}$  is given by

$$D_{01} = D_0 \exp(-E_D/RT) \quad (8)$$

The expression for  $V_f/\omega$  is

$$V_f/\omega = (K_{11}/\omega)W_1[(c_2)_1 + T - (T_g)_1] + (K_{12}/\omega)W_2[(c_2)_2 + T - (T_g)_2] \quad (9)$$

and

$$\omega V_1(0)/K_{11} = 2.303(c_1)_1(c_2)_1, \quad (10)$$

$$\omega V_2(0)/K_{12} = 2.303(c_1)_2(c_2)_2. \quad (11)$$

Maxwell's equation<sup>18</sup> was used to find the effective diffusion coefficient in the composite,  $D_c$ , for the condition where the diffusion coefficient in the dispersed phase is zero.

$$\frac{D_c}{D} = 1 - \frac{3V_c}{V_c + 2} \quad (12)$$

The vapor pressure of the monomer at the outer surface is assumed to be zero because, at this surface, monomer is removed by a fast-flowing sweep of inert gas in the practical context. In previous work, the effect of transport through the porous outer layer has been taken into account;<sup>13,19</sup> but, if the particles are not ultrafine, this resistance is small compared with diffusion in the polymer-containing core and was neglected in the present work.

The concentration of monomer at the center is calculated at each time increment on the linear rate of heating,  $Z$ , from which  $P_1$ , the vapor pressure of monomer over that solution, can be obtained from the Clausius–Clapeyron equation and the Flory–Huggins equation. Thus

$$P_1 = \exp\left(\frac{-\Delta H_v}{RT} + i\right)\theta_1 \exp[(1 - \theta_1) + \chi(1 - \theta_1)^2]. \quad (13)$$

If this pressure rises above ambient, boiling was considered to have occurred, and a lower heating

**Table I Fixed Parameters Used in the Computation (See Ref. 10)**

Symbol	Variable	Value	Units
$(C_1)_1$	WLF parameter for monomer	16.19	—
$(C_2)_1$	WLF parameter for monomer	13.27	K
$(C_1)_2$	WLF parameter for polymer	13.7	—
$(C_2)_2$	WLF parameter for polymer	49.3	K
$K_{11}/\omega$	Free volume parameter for monomer	$1.756 \times 10^{-6}$	$\text{m}^3 \text{kg}^{-1} \text{K}^{-1}$
$K_{12}/\omega$	Free volume parameter for polymer	$5.127 \times 10^{-7}$	$\text{m}^3 \text{kg}^{-1} \text{K}^{-1}$
$(T_g)_1$	Glass transition temperature for monomer	120	K
$(T_g)_2$	Glass transition temperature for polymer	442	K
$V_1(0)$	Monomer specific volume at 0 K	$8.686 \times 10^{-4}$	$\text{m}^3 \text{kg}^{-1}$
$V_2(0)$	Polymer specific volume at 0 K	$7.975 \times 10^{-4}$	$\text{m}^3 \text{kg}^{-1}$
$\xi$	Overlap factor	0.54	—
$\chi$	Flory–Huggins interaction parameter	0.361	—

WLF, Williams–Landel–Ferry.

rate was tested until a critical rate, designed  $Z_c$ , was found that allows the polymer to degrade while keeping  $P_1$  less than ambient throughout.

### NUMERICAL ANALYSIS

The computer program searches for the critical heating rate,  $Z_c$ , at which the peak vapor pressure at the center of the cylinder, throughout the heating schedule, stays just below the ambient pressure. The critical rate,  $Z_c$ , was sought for various combinations of the parameters  $D_0$ ,  $E_D$ ,  $K_0$ ,  $E$ ,  $\Delta H_v$ , and  $i$  that are, in principle, amenable to some degree of control when considering the intelligent design of copolymers.

In all computations, a cylinder of radius  $r_0 = 2.5$  mm was considered, and the volume fraction of the inert filler,  $V_c$ , was taken to be 0.5. The values of all other constants and parameter values occurring in the model equations in the previous section are given in Table I.

Table II contains the five discrete values assigned at some stage of the computation to the six parameters:  $D_0$ ,  $E_D$ ,  $K_0$ ,  $E$ ,  $\Delta H_v$ , and  $i$ . The

center column gives the data used by Evans and colleagues<sup>10</sup> that corresponds to the model system poly( $\alpha$ -methylstyrene), which has been used to test the model experimentally. Temperature was varied in the range  $393 \text{ K} \leq T \leq 1273 \text{ K}$ , and data were considered unrealistic when  $h(T = 393 \text{ K}) < 0.99$ , because it implies that the polymer decomposes at temperatures below 393 K. Computations were truncated if  $Z_c < 0.01 \text{ K h}^{-1}$  (in which case  $Z_c$  was taken as  $0 \text{ K h}^{-1}$ ) or if  $Z_c > 1000 \text{ K h}^{-1}$  (in which case  $Z_c$  was plotted as  $1000 \text{ K h}^{-1}$ ).

The critical heating rates were calculated as functions of the pairs  $(E_D, D_0)$ ,  $(\Delta H_v, i)$ , and  $(K_0, E)$ . For all pairs, each one of the four remaining parameters assumed, in turn, its five values given in Table II, whereas the other three remained fixed at the  $\alpha$ -methylstyrene default values given in the center column of Table II. Clearly, for each of the three pairs, there are four sets of five profiles.

### RESULTS AND DISCUSSION

The parameters selected for combined analysis are those that emerged as having the greatest

**Table II Ranges for the Variable Parameters (Center Column Gives the Default Values)**

Variable	Values					Units
$D_0$	$6.92 \times 10^{-5}$	$3.46 \times 10^{-4}$	$6.92 \times 10^{-4}$	$3.46 \times 10^{-3}$	$6.92 \times 10^{-3}$	$\text{m}^2 \text{s}^{-1}$
$E_D$	$28.37 \times 10^3$	$33.37 \times 10^3$	$38.37 \times 10^3$	$43.37 \times 10^3$	$48.37 \times 10^3$	$\text{J mol}^{-1}$
$K_0$	$1.67 \times 10^{15}$	$8.35 \times 10^{15}$	$1.67 \times 10^{16}$	$8.35 \times 10^{16}$	$1.67 \times 10^{17}$	$\text{s}^{-1}$
$E$	$122 \times 10^3$	$172 \times 10^3$	$222 \times 10^3$	$272 \times 10^3$	$322 \times 10^3$	$\text{J mol}^{-1}$
$\Delta H_v$	$18.94 \times 10^3$	$28.94 \times 10^3$	$38.94 \times 10^3$	$48.94 \times 10^3$	$58.94 \times 10^3$	$\text{J mol}^{-1}$
$i$	21.255	21.755	22.255	22.755	23.255	ln (Pa)

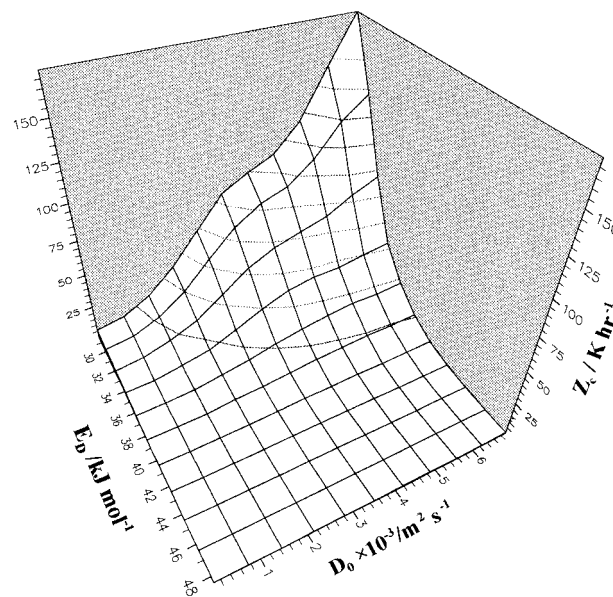
influence on the critical heating rate when varied individually within the range of values that they could reasonably expect to take based on common polymers.<sup>16</sup> In that analysis, the remaining parameters were held at their default value, so the result conceals their combined effects on  $Z_c$ . The problem with such analyses lies not in their execution, but in the interpretation of results. Thus, it is possible to discover that a combination of very high  $\Delta H_v$  and high  $D_0$  gives rise to values of  $Z_c$ , which are well in excess of 1000 K/h, but are unrealizable for the obvious reason that the high molecular weight of degradation product precludes a high value of  $D_0$ .

The commanding parameters so selected fall neatly into three groups. First are those that influence the transport properties, namely pre-exponential coefficient for diffusion,  $D_0$ , and activation energy for diffusion  $E_d$  [eq. (8)]. The latter influences the temperature dependence of diffusion; a low activation energy means that the diffusion coefficient is relatively high in the low temperature region. Second, there are those that control the temperature range over which thermal degradation occurs, namely the specific rate constant  $K_0$  and the activation energy for thermal degradation,  $E$ . The final pair of parameters influences the relationship between monomer concentration and boiling point, namely  $\Delta H_v$  and  $i$ , the pre-exponential coefficient in the Clausius-Clapeyron equation.

### Effect of Transport Properties

In the initial part of the study, the pre-exponential diffusion coefficient  $D_0$  was varied with the activation energy for diffusion  $E_D$  over a range of values of the specific rate constant  $K_0$ . Thus, a series of five surfaces were obtained from plots of  $Z_c$  (vertical axis) against  $E_D$  and  $D_0$  (horizontal axes). Only one such surface is shown in Figure 2, because all surfaces had the same form but different heights. The surfaces show that  $Z_c$  is vanishingly small at low values of  $D_0$  and at high values of activation energy for diffusion. Conversely,  $Z_c$  rises to over 500 K h<sup>-1</sup> at low activation energies and high  $D_0$ . Clearly, such high predicted heating rates, if realizable in practice, would invalidate the assumption that transient heating can be neglected.

The need for high  $D_0$  is transparent from these results, but low  $E_D$  arises because the diffusion coefficient must be reasonably high at low temperatures when thermal degradation is beginning to take place. This result captures the essence of



Peak  $Z_c$  at  $D_0 = 7 \times 10^5 \text{ m}^2 \text{ s}^{-1}$ ,  $E_D = 34 \text{ kJ mol}^{-1}$

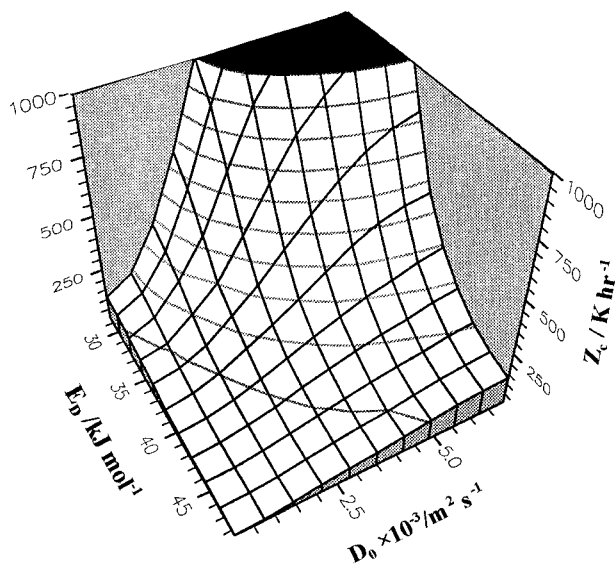
$K_0 / \text{s}^{-1}$	$Z_c / \text{Khr}^{-1}$
$1.67 \times 10^{17}$	175
$8.35 \times 10^{16}$	212
$1.67 \times 10^{16}$	325
$8.35 \times 10^{15}$	390
$1.67 \times 10^{15}$	550

**Figure 2** Effect of systematic variations of  $K_0$  (specific rate constant for thermal degradation) on  $Z_c$  for a basal plane representing transport properties ( $E_D$ - $D_0$ ).

the competition between the rate of generation of monomer and its transport to the free surfaces; both processes are crucially dependent on temperature, although activated by different mechanisms.

Figure 2 also lists the peak value of  $Z_c$  as a function of  $K_0$ , the specific rate constant for the thermal degradation reaction. As  $K_0$  decreases, the corresponding thermogravimetric loss curve shifts to higher temperatures. Thus, the rate of monomer generation peaks at a high temperature, at which the effective diffusant coefficient, itself temperature-dependent, is greater. Thus, a picture, as yet blurred, begins to emerge to show how organic polymers might be deliberately synthesized to make this difficult stage in ceramic processing a rapid and facile step.

The next series of surfaces were generated with the same axes, but for systematic variation of the activation energy for thermal degradation,  $E$ . When this activation energy was low ( $E = 122 \text{ kJ mol}^{-1}$ ),  $Z_c$  was effectively zero for that part of the  $E_D$ - $D_0$  basal plane that was studied.



Peak  $Z_c$  at  $D_0 = 7 \times 10^3 \text{ m}^2 \text{ s}^{-1}$ ,  $E_D = 34 \text{ kJ mol}^{-1}$

$E_D / \text{kJ mol}^{-1}$	$Z_c / \text{K hr}^{-1}$
172	61
222	325
272	875
322	>1000

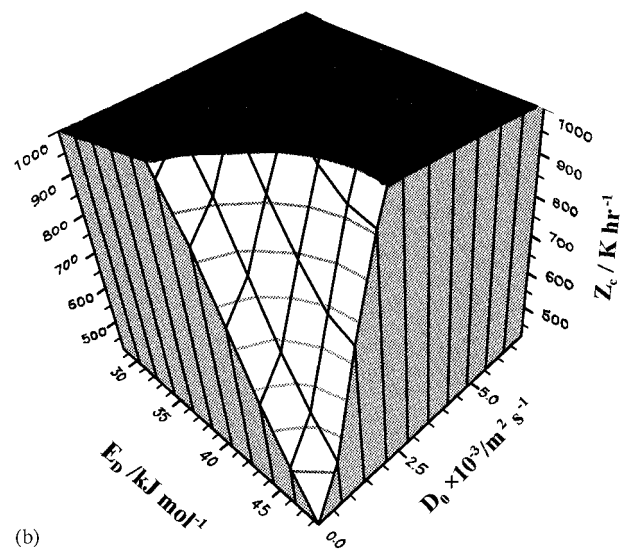
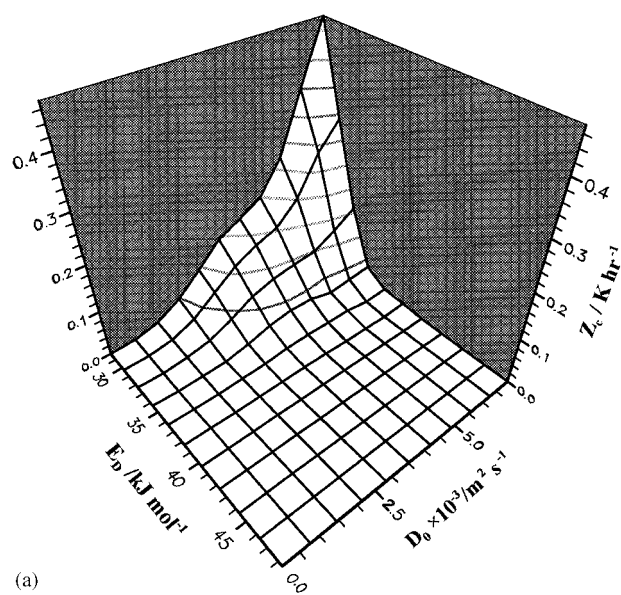
**Figure 3** Effect of systematic variations of  $E$  (activation energy for thermal degradation) on  $Z_c$  for a basal plane representing transport properties ( $E_D$ – $D_0$ ).

In fact, for this value of  $E$ , and for all the values of  $K_0$  studied, degradation takes place below  $100^\circ\text{C}$ . A low activation energy for degradation means that substantial amounts of monomer are being generated in the low temperature regime where the transport to the surface is very slow. Because the monomer so generated cannot escape, the concentration rises very quickly to a high value at the center of the cylinder and boiling ensues. Conversely, when the activation energy for thermal degradation was high (Fig. 3), there was a substantial region over which  $Z_c$  was extremely high. A table is appended to Figure 3 showing the peak values of  $Z_c$  as a function of  $E$ , the form of the surfaces being similar in each case.

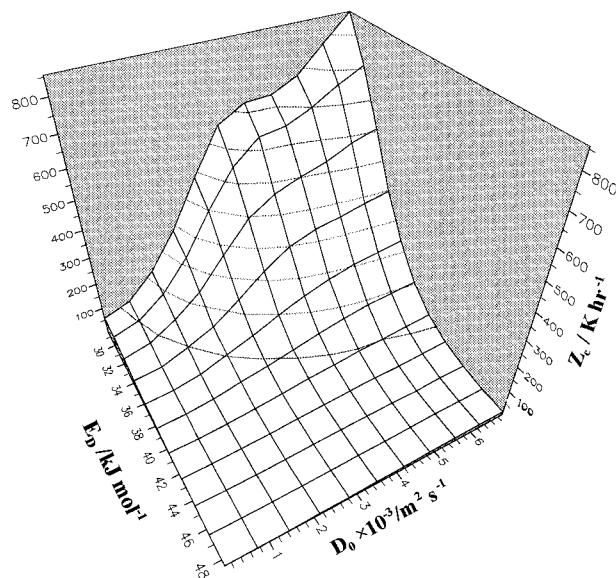
These two parts of the analysis show that the temperature range over which thermal degradation takes place must be matched to the temperature region over which the transport coefficient is high. Although this is an unsurprising result, it has never been taken into account in the selection, yet alone in the deliberate synthesis of polymers for this application. It clearly points to the

selection of a polymer in which the backbone should have a high bond energy, and this in turn influences the nature of side groups that could be appended.

In the next computational experiment, the same horizontal axes representing the transport properties were used, and  $Z_c$  was plotted as ordinate for a range of values of enthalpy of vaporization  $\Delta H_v$ . The enthalpy of vaporization emerges as the single strongest parameter in this study (as shown by Fig. 4). For  $\Delta H_v = 19 \text{ kJ mol}^{-1}$ , all the  $Z_c$  values were  $< 1 \text{ K h}^{-1}$  [Fig. 4(a)], but where  $\Delta H = 59 \text{ kJ mol}^{-1}$  a substantial area of the



**Figure 4** Remarkable effect of  $\Delta H_v$  (enthalpy of vaporization of degradation product) on  $Z_c$  for the basal plane representing transport properties ( $E_D$ – $D_0$ ). (a)  $\Delta H_v = 18.9 \text{ kJ mol}^{-1}$ ; (b)  $\Delta H_v = 58.8 \text{ kJ mol}^{-1}$ .



Peak  $Z_c$  at  $D_0 = 7 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$ ,  $E_D = 34 \text{ kJ mol}^{-1}$

$i / \ln(\text{Pa})$	$Z_c / \text{K hr}^{-1}$
23.26	85
22.76	167
22.26	325
21.76	622
21.26	855

**Figure 5** Effect of systematic variations of  $i$  on  $Z_c$  for a basal plane representing transport properties ( $E_D$ – $D_0$ ). Figure is for  $i = 22$ – $26$ .

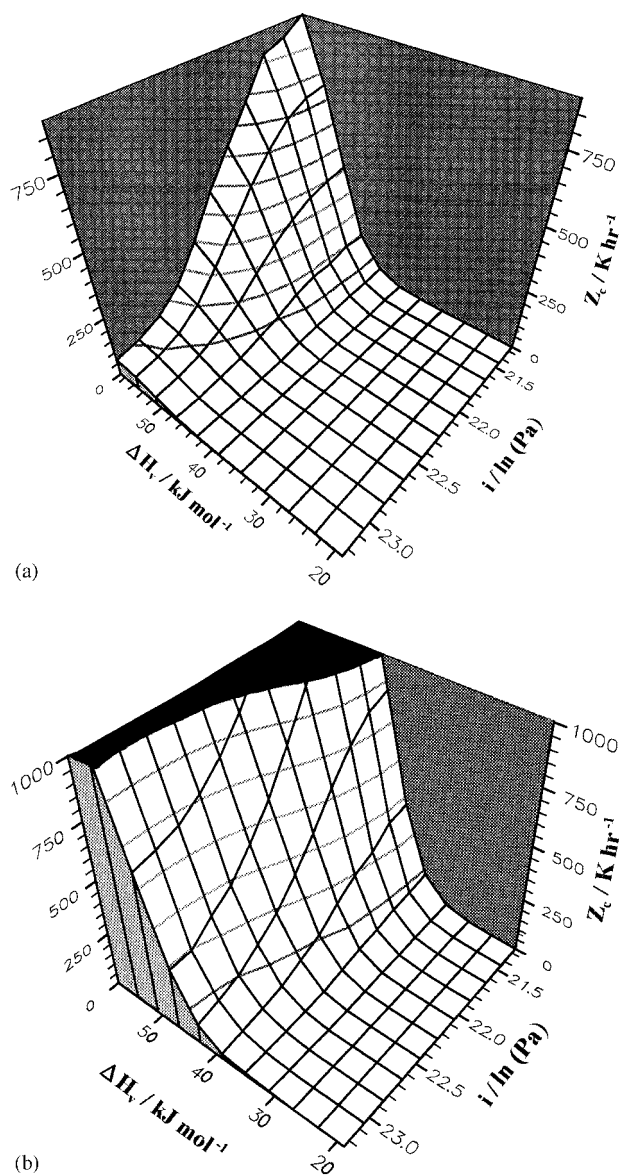
transport parameter space explored presents  $Z_c > 800 \text{ K h}^{-1}$  [Fig. 4(b)]. As discussed previously, such high values of  $Z_c$  violate the assumption that transient heating can be neglected, but this does not deny the prediction of very high values of  $Z_c$ .

Equation (13) shows that, as  $\Delta H_v$  is increased, the vapor pressure is reduced. Conversely, as the pre-exponential coefficient  $i$  is increased, the vapor pressure is increased. Figure 5 shows that the lowest value of  $i = 21.26$  studied gives a peak  $Z_c$  of just over  $800 \text{ K h}^{-1}$ . Its effect is somewhat less strong than that of  $\Delta H_v$ . Of course, there are other parameters that affect the vapor pressure over solution, principally the Florry–Huggins interaction parameter; but, in previous work,<sup>16</sup> they were found to be less influential.

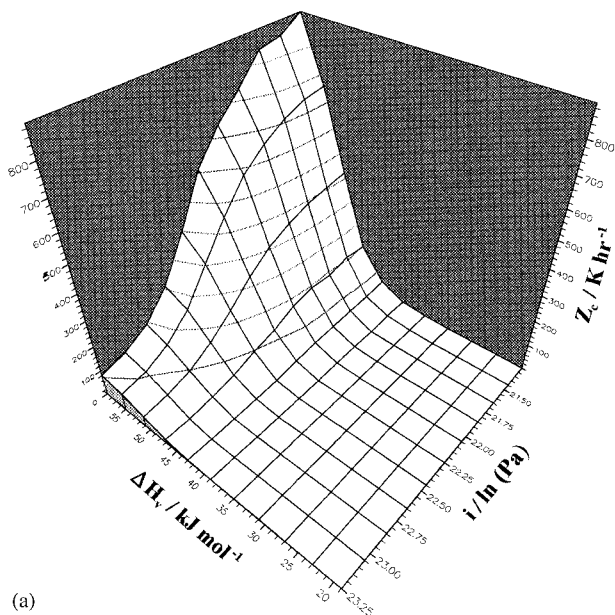
Clearly, the vapor pressure of the degradation product over solution is influenced both by degree of polarity and molecular weight. Thus, a copolymer can be envisaged in which, at intervals, the backbone is weakened either by a conjugated carbon–carbon bond or by the attachment of a side group with high electron affinity. The distance between such groups would determine the fragment size and hence  $\Delta H_v$ .

### Effect of Vapor Pressure Parameters

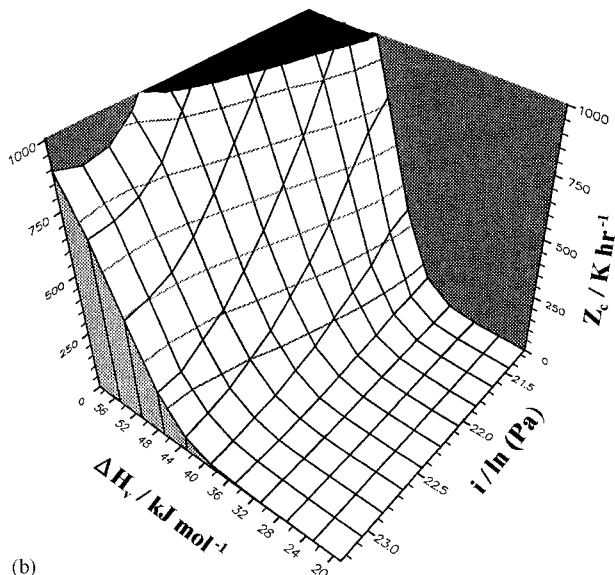
When the parameter space corresponding to the variables that influence vapor pressure was explored, the surfaces generated all showed the steepest gradients, as expected from the overriding influence of  $\Delta H_v$ . Thus, the region of the basal plane corresponding to a high value of  $i$  and a low value of  $\Delta H_v$  consistently gave a negligibly small value of  $Z_c$ . At low values of  $D_0$ ,  $Z_c$  was also low at the end of the plane corresponding to a high value of  $i$  [Fig. 6(a)], but when a high value of  $D_0$  was tested, a high  $Z_c$  could be obtained parallel to the  $i$  axis [Fig. 6(b)].



**Figure 6** Effect of variations in  $D_0$  on  $Z_c$  for the basal plane representing boiling of monomer ( $\Delta H_v$ – $i$ ). (a)  $D_0 = 6.9 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ . (b)  $D_0 = 8.9 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$ .



(a)



(b)

**Figure 7** Effect of variation in activation energy for diffusion  $E_D$ , on  $Z_c$  for the basal plane representing boiling of monomer ( $\Delta H_v-i$ ). (a)  $E_D = 48.4 \text{ kJ mol}^{-1}$ ; (b)  $E_D = 28.4 \text{ kJ mol}^{-1}$ .

Very similar behavior was obtained when the activation energy for diffusion was taken as the third variable. At high values of  $E_D$  [Fig. 7(a)], the surface was very similar to that in Fig. 6(a). High activation energy for diffusion results in a low coefficient of diffusion at low temperatures. Low vapor pressure can nevertheless compensate for this. At high values of  $E_D$ , the influence of  $i$  is removed and Figure 7(b) is very similar to Figure 6(b).

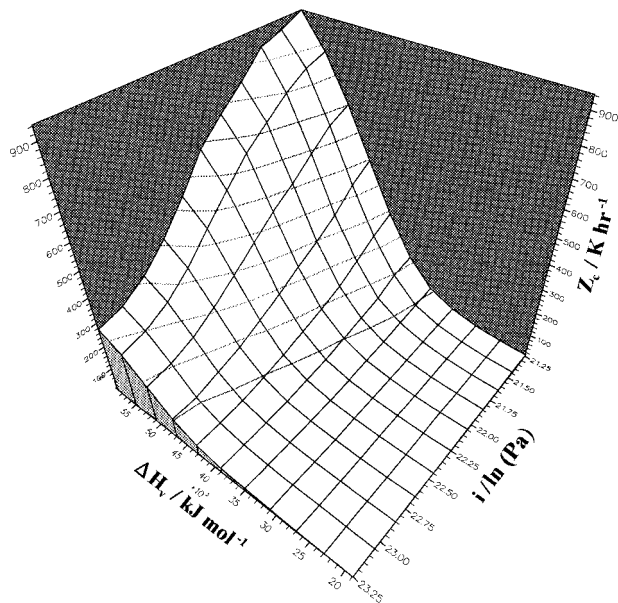
The value of  $K_0$ , the specific rate constant for thermal degradation, had only a limited effect on

the surfaces. A representative surface corresponding to  $K_0 = 1.67 \times 10^{16} \text{ s}^{-1}$  (the midpoint) is shown in Figure 8. The other surfaces differed mainly in the intercept at  $i = 23.25$ ,  $\Delta H_v = 60 \text{ kJ mol}^{-1}$ , which is tabulated in Figure 8.

In contrast, varying  $E$ , the activation energy, for thermal degradation on the vapor pressure basal plane had a dramatic effect on the surfaces. At  $E = 122 \text{ kJ mol}^{-1}$ , the computer program gives unrealistic values because of degradation below  $100^\circ\text{C}$ . At  $E = 172 \text{ kJ mol}^{-1}$  [Fig. 9(a)], about half the basal plane corresponded to  $Z_c$  values that were negligibly small. Where  $E = 322 \text{ kJ mol}^{-1}$ , only the high  $\Delta H_v$ -low  $i$  region of the basal plane offered high  $Z_c$  [Fig. 9(b)].

**Effect of Decomposition Parameters**

Finally, the parameters that influence the thermogravimetric curve,  $K_0$  and  $E$ , were plotted as the basal plane. Figure 10 shows the effect of

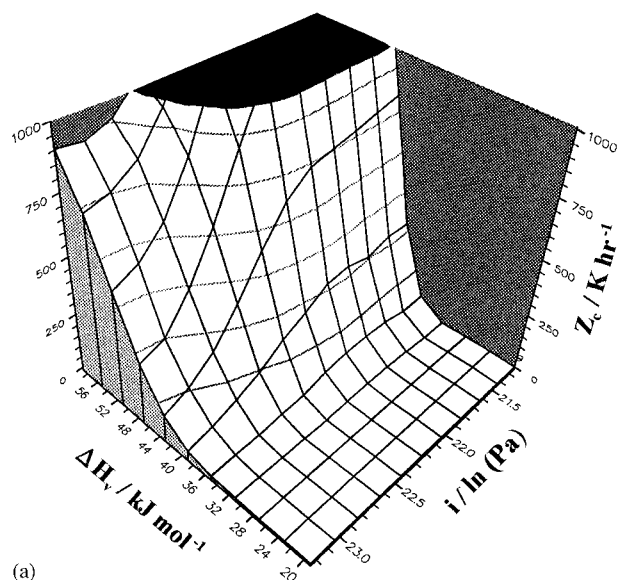


Peak  $Z_c$  at  $i=23.25 \text{ ln (Pa)}$ ,  $\Delta H_v=60 \text{ kJmol}^{-1}$

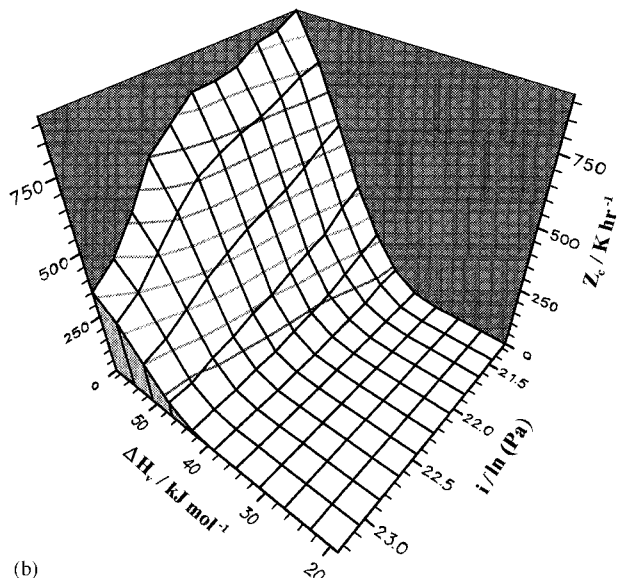
$K_0 / \text{s}^{-1}$	$Z_c / \text{Khr}^{-1}$
$1.67 \times 10^{15}$	260
$8.35 \times 10^{15}$	305
$1.67 \times 10^{16}$	350
$8.35 \times 10^{16}$	490
$1.67 \times 10^{17}$	585

**Figure 8** Effect of systematic variations in the specific rate constant for thermal degradation  $K_0$  on  $Z_c$  for the basal plane representing boiling of monomer ( $\Delta H_v-i$ ). Figure is for  $K_0 = 1.67 \times 10^{15} \text{ s}^{-1}$ .





(a)



(b)

**Figure 9** Effect of activation energy for thermal degradation on  $Z_c$  for the basal plane representing boiling of monomer ( $\Delta H_v$ - $i$ ). (a)  $E = 172 \text{ kJ mol}^{-1}$ ; (b)  $E = 322 \text{ kJ mol}^{-1}$ .

varying  $D_0$  on the critical heating rate. At low  $D_0$  [Fig. 10(a)], values of  $Z_c$  are low throughout the basal plane. At high  $D_0$  [Fig. 10(b)], a ridge of high  $Z_c$  corresponds to a high activation energy for thermal degradation (decomposition and liberation of products at high temperatures), and the effect of  $K_0$  is secondary.

This behavior is also seen when  $E_D$  is varied systematically (Fig. 11). A combination of low  $E_D$  and high  $E$  consistently gives high critical heating rate. The high  $E$  means that decomposition occurs in the high temperature regime, and low

$E_D$  means that the diffusion coefficient is high over a wide temperature range.

Predictably, when the value of  $Z_c$  on this basal plane is explored for different values of  $\Delta H_v$ , the effect is dramatic. Figure 12(a) shows the effect of low  $\Delta H_v$  (a degradation product with a low boiling point) that gives very low values of  $Z_c$  throughout the plane. When  $\Delta H_v$  is high [Fig. 12(b)], high  $Z_c$  is achieved across the basal plane, except for  $E = 122 \text{ kJ mol}^{-1}$ , where the combination of  $E$  and  $K_0$  gives a thermogravimetric curve for which degradation is complete below  $100^\circ\text{C}$ . This is unlikely to represent a real system, and the model does not accommodate diffusion calculated below  $100^\circ\text{C}$ , because diffusion is so low that  $Z_c$  is zero. The effect of  $i$  is similar to that of  $\Delta H_v$ , but less pronounced; a high  $i$  corresponds to low  $Z_c$ .

## CONSEQUENCES FOR MATERIALS SELECTION AND SYNTHESIS

In principle, the analysis presented herein provides a foundation for the deliberate synthesis of polymers that will enable a ceramic body to survive the critical initial stage of thermolysis without disrupting the assembly of particles. At a later stage, continuous porosity develops and gas transport assists the displacement of organic matter. In practice, the realization of this goal is complicated because the ranges of values for the six parameters studied place conflicting demands on bonding and on structure.

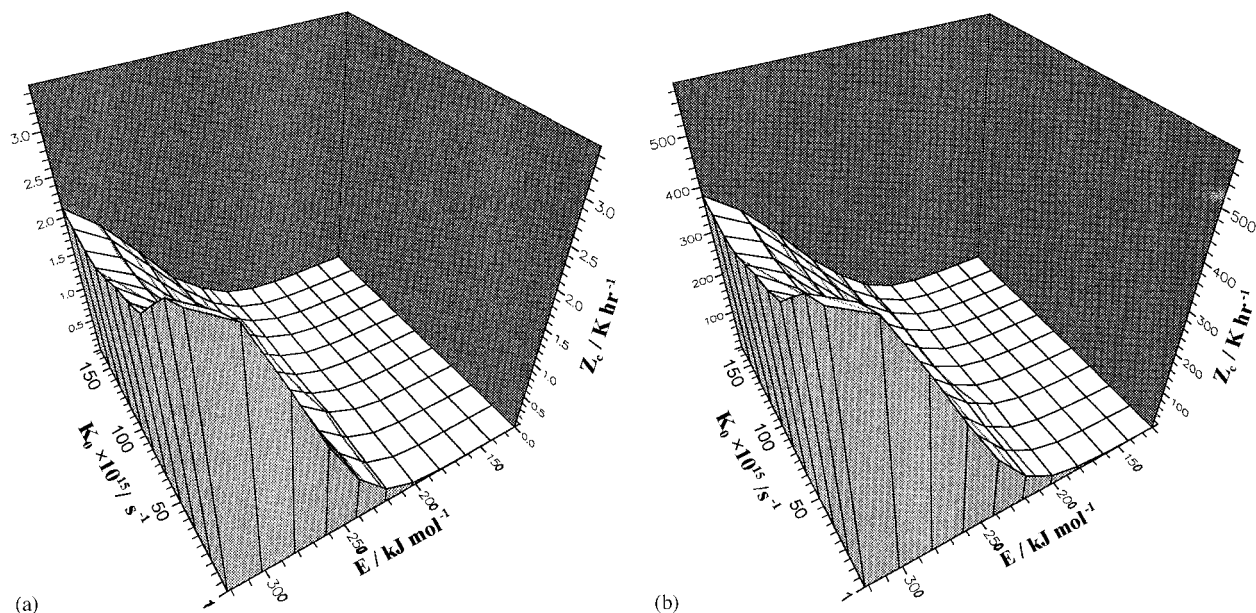
In many ways, the simplest variable to address is the enthalpy of vaporization of the degradation product. There have been many attempts to relate  $\Delta H_v$  to other parameters.<sup>20-22</sup> The enthalpy itself depends on temperature, and Yaws and Yang<sup>20</sup> have correlated  $\Delta H_v$  at a temperature  $T$  to  $\Delta H_v^1$  at the normal boiling point  $T_1$  and to the critical temperature  $T_c$  by

$$\Delta H_v = \Delta H_v^1 \left( \frac{T_c - T}{T_c - T_1} \right)^{0.38} \quad (14)$$

for 700 organic liquids. Perhaps one of the most useful surveys for this purpose is by Nikolaev and coworkers,<sup>21</sup> in which  $\Delta H_v$  is given by

$$\Delta H_v = f \left[ I \frac{m}{d} \frac{(n^2 - 1)}{(n^2 - 2)} \right] \quad (15)$$

where  $I$  is the ionization potential,  $m$  is the molecular weight,  $d$  is the density, and  $n$  is the



Peak  $Z_c$  at  $K_0 = 1.67 \times 10^{15} \text{ s}^{-1}$ ,  $E = 322 \text{ kJ mol}^{-1}$

$D_0 / \text{m}^2 \text{ s}^{-1}$	$Z_c / \text{K hr}^{-1}$
$6.92 \times 10^{-5}$	4
$3.46 \times 10^{-4}$	25
$6.92 \times 10^{-4}$	54
$3.46 \times 10^{-3}$	321
$6.92 \times 10^{-3}$	692

**Figure 10** Effect of  $D_0$  on  $Z_c$  for the basal plane representing thermal degradation ( $K_0$ – $E$ ). (a)  $D_0 = 6.92 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ . (b)  $D_0 = 6.92 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$ .

refractive index. When  $\Delta H_v$  is plotted against the function on the rhs of eq.(15) for a selection of aromatic and aliphatic hydrocarbons, it gives an average slope of  $2.8 \times 10^{27} \text{ m}^{-3}$  and intercept at  $\Delta H_v = RT$  ( $2.48 \text{ kJ mol}^{-1}$  at  $298 \text{ K}$ ). There is a slight spread in the results that corresponds to a lower slope for aromatic or cyclic hydrocarbons, compared with that for aliphatics. A comparable plot for polar liquids gives slopes of  $4.6 \times 10^{27} \text{ m}^{-3}$  and  $3.0 \times 10^{27} \text{ m}^{-3}$  for aliphatic and aromatic compounds, respectively.

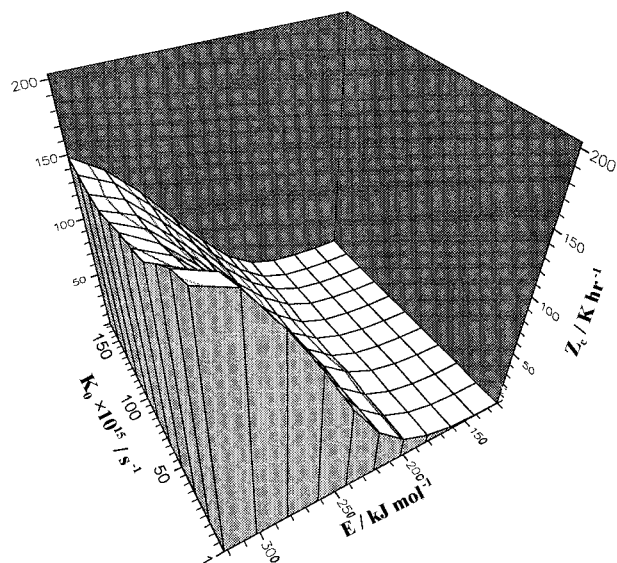
Thus, for the same value of the function of molecular weight, density, refractive index, and ionization potential given in eq. (15), the enthalpy of vaporization follows the following trend: aromatic and cyclic hydrocarbons < aliphatic hydrocarbons < polar aromatics < polar aliphatics. Ionization energies for a wide range of organic molecules are tabulated.<sup>23</sup> The effect of molecular weight is shown by a range of expressions for vapor pressure<sup>24</sup> and provides the main route to control of  $\Delta H_v$  through fragment size.

Thus, the degradation product molecular weight could, in principle, be controlled by the judicious

selection of carbon—carbon bonds in the backbone interspersed with weak units or with bonds weakened by side groups with high electron affinity placed at intervals.

Activation energy for thermal degradation  $E$  is clearly related to bond energy, and the predicted desire for high  $E$  may conflict with a deliberately weakened chain at selected sites. The literature gives very few values of  $K_0$ , but clearly a low value of  $K_0$  corresponds to a high stability polymer. This is achieved in engineering polymers through the use of recurring aromatic groups and ether linkages, but a complicating factor in ceramic processing is that the polymer should decompose to leave no carbon residue.

A more serious problem surrounds the acquisition of key transport data  $D_0$  and  $E_D$  that appear in eq. (8), as well as the ratio of solvent and polymer jumping units. A limited number of systems have been analyzed using this free volume theory and, for those that have, available data are tabulated in Table III. In later assessments of the Vrentas–Duda theory,<sup>25,26</sup> the activation energy for diffusion  $E_D$  is considered to be concentration-



Peak  $Z_c$  at  $K_0 = 1.67 \times 10^{15} \text{ s}^{-1}$ ,  $E = 322 \text{ kJ mol}^{-1}$

$E_D / \text{kJ mol}^{-1}$	$Z_c / \text{K hr}^{-1}$
28.37	226
33.37	111
38.37	54
43.37	25
48.37	11

**Figure 11** Effect of activation energy for diffusion  $E_D$  on  $Z_c$  for the basal plane representing thermal degradation ( $K_0$ - $E$ ),  $E_D = 28.4 \text{ kJ mol}^{-1}$ .

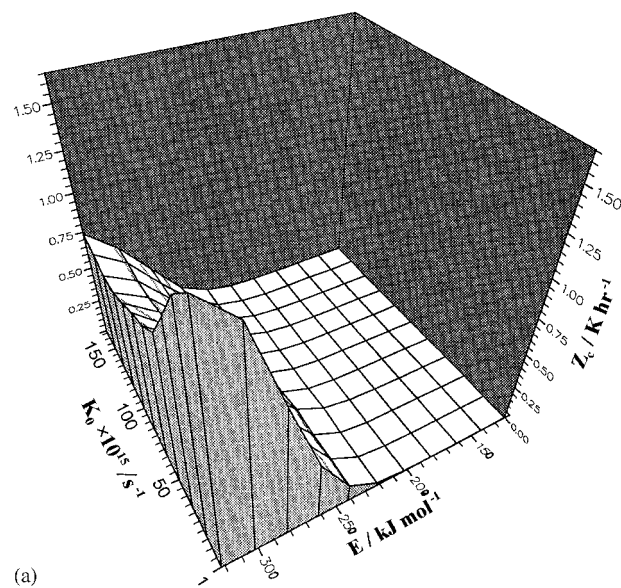
dependent, and estimates of the remaining parameters  $D_0$  and  $\xi$  are based on the assumption  $E_D = 0$  at all concentrations. This does, however, contribute to the deviation of predictions from experimental data.<sup>25</sup>

$D_0$  is considered to be a solvent-dominated parameter, and this is confirmed by experimental measurements in which  $D_0$  emerges as identical for the same diffusant in different polymers. Treating it as a solvent property opens the door to the estimation of all the parameters in the free volume equation without reference to solvent polymer diffusion data.<sup>25</sup>

## CONCLUSIONS

This exploration of the combined effects of the three sets of parameters that influence the formation of defects in the thermolysis of ceramic moldings provides a set of criteria for intelligent polymer synthesis. Consistently high critical rates are obtained for a low activation energy for diffusion and a high pre-exponential  $D_0$ . Values of param-

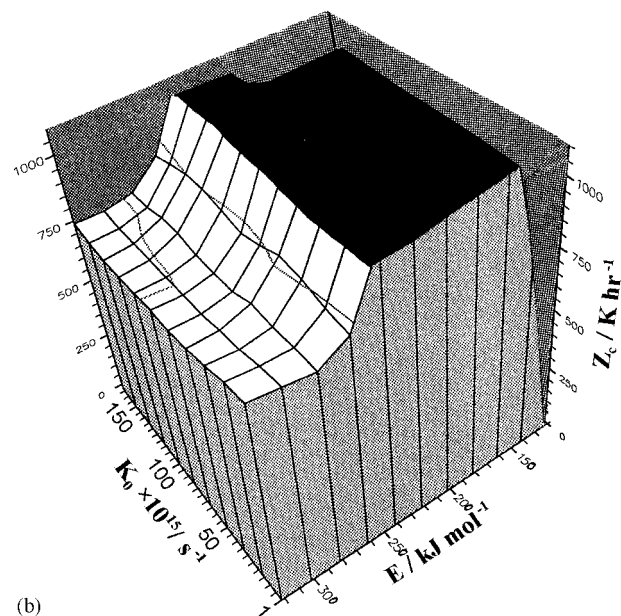
eters controlling polymer decomposition are preferred if they produce onset of decomposition at a high temperature, corresponding to the tempera-



(a)

Peak  $Z_c$  at  $K_0 = 1.67 \times 10^{15} \text{ s}^{-1}$ ,  $E = 322 \text{ kJ mol}^{-1}$

$\Delta H_v / \text{kJ mol}^{-1}$	$Z_c / \text{K hr}^{-1}$
19	2
29	11
39	54
49	227
59	882



(b)

**Figure 12** Effect of  $\Delta H_v$  on  $Z_c$  for the basal plane representing thermal degradation ( $K_0$ - $E$ ). (a)  $\Delta H_v = 18.9 \text{ kJ mol}^{-1}$ ; (b)  $\Delta H_v = 58.9 \text{ kJ mol}^{-1}$ .

**Table III Values of Diffusion Parameters for Polymer-Solvent Systems**

Polymer	Solvent	$D_o$ ( $m^2 s^{-1}$ )	$\xi$	$E_D$ (kJ mol $^{-1}$ )	Source
Polystyrene	Toluene	$6.15 \times 10^{-6}$	0.53	22	17
Poly(methylacrylate)	Methyl acetate	$8.71 \times 10^{-6}$	0.57	15	17
Polystyrene	Ethylbenzene	$6.92 \times 10^{-6}$	0.56	38	17
Polystyrene	Toluene	$4.82 \times 10^{-6}$	0.58	0	25
Poly(vinyl acetate)	Toluene	$4.82 \times 10^{-6}$	0.82	0	25
Polystyrene	Ethylbenzene	$4.61 \times 10^{-6}$	0.69	0	25

ture region under which mass transport rates are high. The enthalpy of vaporization emerges as an overwhelmingly influential variable in all the calculations performed, often able to compensate for unfavorable values of other parameters.

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#### NOMENCLATURE

$\dot{q}$	rate of production of monomer based on mass of polymer	$s^{-1}$
$\dot{Q}$	rate of production of monomer based on total volume of suspension	$kg m^{-3} s^{-1}$
$R$	gas constant	$J mol^{-1} K^{-1}$
$r$	radius	m
$r_0$	radius of cylindrical molding	m
$r_n$	radius of the polymer-containing core	m
$C$	concentration of monomer in solution in the polymer phase based on the total volume of suspension	$kg m^{-3}$
$D$	diffusion coefficient in polymer	$m^2 s^{-1}$
$D_c$	diffusion coefficient in suspension	$m^2 s^{-1}$
$D_o$	pre-exponential factor in eq. (8)	$m^2 s^{-1}$
$d$	density	$kg m^{-3}$
$E_D$	activation energy for diffusion	$J mol^{-1}$
$E$	activation energy for thermal degradation	$J mol^{-1}$
$\Delta H_v$	enthalpy of vaporization	$J mol^{-1}$
$\Delta H_v^1$	enthalpy of vaporization at the boiling point	$J mol^{-1}$
$h$	fraction of mass of polymer remaining	$0 < h < 1$
$I$	ionization potential	J
$i$	constant in eq. (13)	ln (Pa)
$K_{11}$	free volume parameter for monomer	$m^3 kg^{-1} K^{-1}$
$K_{12}$	free volume parameter for polymer	$m^3 kg^{-1} K^{-1}$
$m$	molecular weight	$kg m L^{-1}$
$n$	refractive index	—
$P_1$	vapor pressure of monomer over solution	Pa
$T$	temperature	K
$T_1$	boiling point	K
$T_c$	critical temperature	K
$(T_g)_1$	glass transition temperature of monomer	K
$(T_g)_2$	glass transition temperature of polymer	K
$V_f$	average hole-free volume per unit mass	$m^3 kg^{-1}$
$V_1(0)$	specific volume of monomer at 0 K	$m^3 kg^{-1}$
$V_2(0)$	specific volume of polymer at 0 K	$m^3 kg^{-1}$
$V_c$	powder volume loading	$0 < V_c < 1$
$W_1$	weight fraction of polymer in polymer-monomer solution	$0 < W_1 < 1$
$W_2$	weight fraction of polymer in polymer-monomer solution	$0 < W_2 < 1$
$Z$	heating rate	$K s^{-1}$
$Z_c$	critical heating rate	$K s^{-1}$
$\phi$	volume fraction of monomer in polymer	$0 < \phi < 1$
$\chi$	interaction parameter for polymer-monomer system	—
$\omega$	overlap factor for free volume	—

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